Surprising Harmonies

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> "… The tenacity of traditions yields an unexpected advantage. It is only where expectations are formed that they can also be reassuringly conformed, playfully disappointed or grandly surpassed."
>
> Ernst Gombrich, *The Sense of Order*.

Abstract:

Understanding surprise is a key to the cognition of music, at all levels of musical structure: rhythm, melody, harmony, timbre. This paper addresses the modeling of surprise in particular music sequences: Jazz harmonic progressions. Most of the works in music cognition relate surprise to the phenomenon of musical expectation: a surprise is seen as something unexpected. Furthermore, unexpected more or less means "unheard before". In this paper, we emphasize the importance of the rich algebraic structure underlying Jazz chord sequences, and suggest that harmonic surprise may not only be related to unexpected structures, but also to "calculus", i.e. to an ability to deduce a sequence from a set of combinatorial rules. We first introduce the domain of Jazz chord sequences and describe its underlying algebraic structure, based on the notion of *chord substitution*. We then propose to use a statistical-based data compression approach to infer recurring patterns from the corpus, and show that this yields reasonable but limited expectation structures. We then propose a mechanism to induce chord substitution rules from the corpus, and comment its output according to the theory of chord substitution. Finally, we suggest that such a model of chord substitution rules may be used to devise richer models of harmonic surprise.

Keywords: models of expectation, models of surprise, unsupervised learning of musical structure, Jazz harmony, rewriting rules

1. Introduction

Repetition is often pointed out as the main process governing music production and perception: "Repetition breeds content", as the proverb says, and experimental psychology has long shown the importance of repetition in musical cognition, since the early stages of musical development (Deliege & Sloboda, 1995). However, purely repetitive music also brings boredom, and surprise plays as central a role in musical perception as repetition. It is probably our ability and desire to be surprised that drives us to listen to music, and also pushes us to discover new musical styles. Of course, it is probable that composers as well as listeners look

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for some compromise between repetition and surprise, as suggested nicely by (Smith & Melara, 1990): ".. maximum aesthetic pleasure arises when music is optimally discrepant from a schematic ideal, with musical events moderately assimilable, and moderately difficult to comprehend. In this view, aesthetic pleasure comes from an exquisite game of expectational cat and mouse with the composer, in which the listener enjoys the tensions and the resolutions, the problems posed and the problems solved, the confusions followed by comprehension". The experiments conducted by the authors demonstrate indeed that deviance from prototypicality influence aesthetic judgement made by listeners, and that there seems to be, at least for particular groups of listeners, such an "ideal" position between prototypicality and deviance, similarity and difference, or, in our view, repetition and surprise.

In this context we argue that understanding surprise is a key to musical cognition. Surprise may occur at all levels of musical structures: rhythm, melody, harmony, and even timbre. Following Gombrich (1984), we believe that for an "interesting" or "exciting" surprise to occur, there needs to be strong expectations built and deceived. These strong expectations are themselves the result of long exposure to musical material in a given style. The goal of the present study is to model the mechanisms by which expectations are created, fulfilled, disappointed or surpassed, and therefore surprise can be achieved.

There seems indeed to be a consensus concerning musical surprise in that surprise is more or less taken as an equivalent to "unexpected". This explains probably why numerous studies have been conducted on modeling anticipation and expectation in musical cognition (Bharucha, 1987, Narmour, 1992).

In this paper we focus on the harmonic dimension of music, without committing to other dimensions of music perception, and focus on the corpus of Jazz music, because we believe its characteristic combinatorial aspect makes standard approaches in music cognition not adapted. We believe that here is something specific to harmonic surprise - particularly in tonal music - because harmony involves high-level combinatorial structures. Combinatorial properties of music have been studied from a purely compositional and mathematical viewpoint by several researchers (Allouche, 1995; Chemillier and Timis, 1988). The impact of combinatorial structures on music perception is, however, less understood.

Experimental psychology shows that harmonic context plays a crucial role in the perception of musical sequences (Bigand and Pineau, 1997; Drake, 1998). In this context, (Eberlein, 1995) suggests to use a statistical-based approach to model the gradual learning of harmonic successions, based on an improved and neutral system of harmony description. Similarly, various connectionist models (Page, 1994; Leman, 1995) have been proposed to simulate the learning of a representation of harmonic knowledge. Several authors have tried in particular to model this phenomenon at the melodic and harmonic level. For instance, the MUSACT framework (Bharucha, 1987) provides a connectionist model of harmonic expectation. Tillmann and Barucha (1998) further show that their system converges to an end-state with self-organization which corresponds to the rules of classical harmony. These works prove that it is possible to learn automatically the building up of harmonic expectancies over time from passive exposure to music sequences.

However, the corpus used in the studies - Classical four-part music - is based on a pure model of classical harmony involving only triads, i.e. simple 3-note chords. It is not clear how such a connectionist approach can scale up and be applied to account for the much more complex harmonic structures found in Jazz music, as outlined in the next section. Indeed, although Jazz harmony comes from Classical harmony from an evolutionary viewpoint, we argue that the harmonic functions of chords are much more complex than in Classical four-part chorals, because of the underlying combinatorial "game" at play. To take a simple example, in the context of `C` major, these theories would consider a `F#` chord as the most "distant" possible tonal context (Tillmann & Barucha, 1998). In Jazz however, a `C(7)` and `F#(7)` are closely related, and may even be considered interchangeable. Another distinction made in Classical four-part choral music is between a pure `C` major chord (playing a role of, say, a stable first degree) and a `C 7` chord (playing the role of an unstable dominant seventh tending to resolve to `F`). In Jazz also, `C` and `C7` are often considered equivalent, and the difference made by Classical music is somewhat blurred.

Using universal information theoretic approaches, (Dubnov et al., 1998) classify melodies (Midi files) by computing a similarity distance based on cross entropy. The approach is validated by showing that the resulting automatic classification concords almost exactly with the usual classification of musical styles.

In this paper, we emphasize the importance of taking into account the rich algebraic and combinatorial structure underlying Jazz chord sequences, and suggest that harmonic surprises in this context may be measured in accordance with this structure. Our intuitive idea is that harmonic surprise is related to our ability to "understand" chord sequences, and that this ability may be faithfully represented by two main ingredients. First a set of recurring *patterns*, which can be seen as a signature of the underlying musical style, and which are the basis of expectation structures. Second, the ability to transform these patterns, according to a set of *substitution rules.* These rules allow to extend drastically the amount of possible patterns and create many different "acceptable" musical data out of a compact set of rules.

We will basically follow the information-theoretic approach of (Dubnov et al., 1998), because of its simplicity and efficiency. Section 3 describes briefly the mechanism and the extension to take into account the specifics of chords (chord structure and chord transposition in particular) to compute a model of harmonic expectation. We then show that this model faithfully represents expectation structures of Jazz harmony, and can be used to yield a reasonable but limited notion of surprise based on these expectation structures. In Section 4 we show how to induce rules automatically by gradual learning, and compare the results to the theory. We finally conclude on the possibility given by these methods to better represent surprise in Jazz harmony.

2. The Algebra of Jazz Chord Sequences

This section introduces the domain of chord sequences, as our main object of study. Moreover, we focus on a specific musical style, Jazz, where chord sequences play a particularly important role. Indeed, in the context of Jazz improvisation, chords are often considered as even more important than the actual notes of the musical piece.

2.1 Jazz Chord Sequences

Jazz chord sequences are not just any sequences of arbitrary chords. Musicologists have tried to capture the essence of Jazz chord sequences in various ways. One way is to trace back the origin of Jazz to basic musical structures such as the Blues, and then apply the rules of classical harmony to understand how these basic structures have been transformed. These transformations are expressed usually in terms of *chord substitution rules*. A chord substitution rule is a kind of "rewriting rule", which allows to transform any subsequence of chords into another subsequence of chords, which is harmonically equivalent. This transformation allows to introduce diversity, without, in principle, changing the harmonic function of the subsequence. One important aspect of these rules is that they always make sense in terms of classical harmony.

To understand our context, let us take an example. Figure 1 shows a Jazz melody (*Blues for Alice*, by Charlie Parker). On top of the melody, chords are indicated. These chords represent harmonic information and have several roles. First they allow the accompanists (e.g. piano, guitar) to play along, by giving the necessary harmonic information, much in the same way harpsichords back up singers in Baroque Music, using figured bass (Bukovzer, 1947). Second, chords are useful also for the soloist, because they give indications on which scales may be used for improvisation. This second aspect has been the object of several studies, in particular related to the production of improvisation (Järinen, 1995; Johnson-Laird, 1991), or the analysis of chord sequences (Ulrich, 1977; Pachet, 1999).

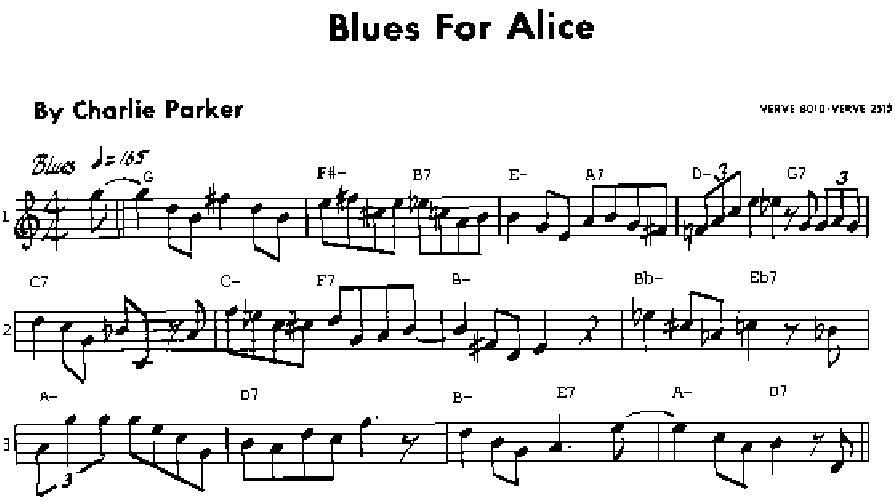

Figure 1. A Jazz tune (Blues for Alice, written by Charlie Parker, in Bb version[2]), backed up by Jazz chords.

In the context of Jazz music, these chords are so important that often, this is the only information shared by the different players. Jazz chord sequences are gathered in well known books, such as the Real Book (1981), the Fake book (1983), or the Charlie Parker Omnibook (1978) which contain about 2000 Jazz chord sequences composed in the 50s or the 60s.

2.2 Notation

For the purpose of this paper, we will use a simple but normalized notation for chord sequences. Chords are represented as a couple {pitch class, structure}. Pitch classes are one of the possible pitch classes (e.g. A, B, C, …, A#, B#, etc.). The structure is a string representing the harmonic content of the chord. The structure allows a musician to infer exactly the list of notes making up the chord. In Jazz this structure may be quite rich and varied. Typical structures are: `min` (a minor chord), maj7 (a major seventh chord, 7 (a dominant seventh chord), aug5 7 9 (a seventh chord with augmented fifth and perfect ninth), and so forth.

Temporal sequences of chords are represented as follows. We assume that we have only 4/4 tunes, and each measure contains either 1, 2, 3 or 4 chords. Temporal information is

[2] © 1956, Atlantic Music Corporation, © renewed 1984 Atlantic Music Corporation. From *The Charlie Parker Omnibook*, Bb version, reprinted with permission of the publisher.

represented by the following separators: "," indicates the separation between the first and second beat, "/" between the second and third beat, ";" between the third and fourth beat. Finally, "|" separates two 4 beat measures.

Using this notation, the chord sequence corresponding to Figure 1 (in C version, i.e. transposed in F) is represented by the first string in Figure 2 , together with examples of typical chord sequences found in these corpuses using our notation.

```
bluesForAlice
F  | E halfDim7 / A 7  | D min / G 7  | C min / F 7  | Bb 7  | Bb min / Eb 7  | A
min | Ab min / Db 7  | G min 7  | C 7  | F 7 | G min / C 7  |

Marmaduke
G min  | G min  | G min  | G min / C 7  | F  | G min / C 7  | F  | A min / D 7  | G
min  | G min  | G min  | G min / C 7  | F  | G min / C 7  | F  | F  | C min  | F 7
| Bb  | Bb  | G 7  | G 7  | G min  | C 7  | G min  | G min  | G min  | G min / C 7
| F  | G min / C 7  | F  | A min / D 7  |

NowsTheTime
F 7  | F 7  | F 7  | F 7  | Bb 7  | Bb 7 | F 7  | D 7  | G min  | C 7  | F 7  | C 7
|

ornithology
G  | G  | G min  | C 7  | F  | F  | F min  | Bb 7  | Eb 7  | A halfDim7 / D 7  | G
min  | D 7 aug9  | B min  | E 7  | A min  | D 7  | G  | G  | G min  | C 7  | F  | F
| F min  | Bb 7  | Eb 7  | A halfDim7 / D 7  |  G  | G  | B min / E 7  | A min / D
7  | G / E 7  | A min / D 7  |
```

Figure 2 Examples of Jazz chord sequences (all by Charlie Parker).

The purpose of the paper is to study these kinds of sequences, and to show how musical expectation and surprise may be built up from gradual listening of these sequences. To understand how expectation and surprise may pop up from this background, we will explain briefly how chords work in the next section.

2.3 Patterns of Chord Sequences

Jazz Chord sequences exhibit regularities which are well known by Jazz musicians. These regularities create deep expectations of continuations. Many of these regularities come from classical music, and are governed by the mechanism of resolution: a seventh chord creates an expectation of its resolution. This expectation is even stronger when the seventh chord is duly prepared. For instance, a sequence such as : `C/ A min 7 | D min 7 / G 7` will most probably create an expectation of a C major chord to occur next, in a trained western tonal ear.

Additionally, Jazz music also includes lots of musical structures of its own. For instance, the famous "two-five-one" structure indicates a sequence of three chords such as (D min 7 / G 7 | C) which is typical of Jazz standards. So-called *turnarounds* such as (`C / A 7 | D min 7 / G 7`) are other examples of typical pattern of chords, usually found at the end of a tune. Tritone pattern such as `C | F#7 | F` are also very frequent in Jazz (much less in Classical music). Many such harmonic patterns have been identified and can be found in almost all textbooks on Jazz harmony.

2.4 Chord Substitution Rules

One important characteristic of Jazz harmony consists in twisting an existing piece to make it sound different, within certain limits, so that it is still recognizable, without being always the same. These twists are often represented (and taught) as a set of substitution rules, and found in almost all books on Jazz harmony, such as (Josefs, 1996). These descriptions are, however, more or less formalized. Steedman (1984) was probably the first author to propose a fully formalized set of substitution rules, in the form of a context-free grammar for the subset of 12-bar Blues sequences.

We give below examples of the most common chord substitutions in the context of Jazz (and also pop music). A common notation for these rules is `left part → right part`, where left part and right part are arbitrary chord sequences. The only constraint is that the two parts of the rule take the same amount of time.

2.4.1 Examples of chord substitution rules

- Repetition

This rule allows any chord to be repeated, as long as the repetition takes the same amount of time than the original chord, i.e. each repeated chord takes half the time.

(Repetition) `C → C / C`

- Enrichment of chords

Jazz music tends to use more complex chords than pure classical music. If is therefore common to replace simple chords by more complex chords, built by adding extra notes to basic chords. For instance, a `C seventh` chord will often be replaced by a more complex `C 7 9 11`. Similarly, a `C minor` chord (which contains only three notes) will be often replaced by a `C min 7 9` (5 notes), when appropriate. Since there are a lot of possible chord enrichments, it is not practical to write them all as rules. An example could be:

(Enrichment) `C 7 → C 9`

- Relative minor

This rule comes from classical harmony, and reflects the equivalence between major and relative minor chords, which share almost the same not set.

(Relative) `C → A min`

- Tritone Substitution

This rule is probably the most characteristic rule of Jazz. It states an equivalence between seventh chords and their tritone. The rule can be explained in terms of classical harmony (although, these two chords are opposed in the circle of fifths, they share the same third and seventh).

(Tritone Substitution) `C 7 → F# 7`

- Preparation

Preparation rules allow one chord to be replaced by two or more chords.

(Preparation by Seventh) `C → G 7 / C`

Here the rule allows any chord to be prepared by its seventh chord. This increases the feeling of progression, without creating new harmony. Another but somewhat equivalent kind of preparation is with a minor seventh chord:

(Preparation by Minor Seventh) `G 7 → D min 7 / G 7`

- Transition to Fourth chord

This rule, proposed by Steedman, introduces fourth chords in sequences. Fourth chords are stable chords which stress the tonality of the replaced chord. The rule can be stated as follows:

(Transition to Fourth) `C 7→ C 7 / F`

- Back propagation of seventh

This is a more complex rule dealing with retro propagation of seventh chords. This phenomenon has been pointed out by Steedman (1984), and appears necessary for building a full grammar of Jazz chord sequences. The rule does not modify the sequence *per se*, but only its temporal structure. It states that a seventh chord can somehow move backwards in time, thereby stressing its role of preparation by anticipating its occurrence:

(Back Propagation of Seventh) `X X C7 Y → X C7 Y Y`

- Left Deletion

Finally, some chords may be occasionally deleted, once again without changing the harmonic content. This is typically the case after the preceding rule has been applied (this shows the difficulty of formalizing in a proper way this chord substitution mechanism):

(Left Deletion of Seventh) `X C7→ X X`

The rule set described here is by no means exhaustive (it is indeed a research issue to exhibit a minimal and complete set of rules which would allow to recreate all Jazz chord sequences and only Jazz chord sequences), nor intended to provide an operational model of a grammar of Jazz chord sequences. It is just an attempt to summarize the most important chord substitution rules needed to create chord sequences in the style of the corpus mentioned above.

An important aspect of these rules is their ability to be combined in a recursive and combinatorial fashion. To illustrate this aspect, we give below examples of typical combinations of these rules for creating complex chord sequences.

2.4.2 Example #1: Chromatic descent from a basic Blues structure

Let us consider the following starting - and simple - sequence (the beginning of a basic Blues):

`C | F | C | C7 | F`

This simple sequence can be modified by using chord substitution to create a much more harmonically interesting sequence (a chromatic descent with alternating minor seventh and seventh chords):

`C | B min 7 / A# 7 | A min 7 / G# 7 | G min 7 / F# 7 | F …`

The rules to be used are the following (rules are applied to bold chords) :

`C | F | C |` **`C7`** `| F …` with (Preparation by Minor Seventh):
`C | F | C | G min 7 /` **`C7`** `| F …` with rule (Tritone Substitution):
`C | F | C |` **`G min 7`** `/ F#7 | F …` with rule (Preparation by Seventh):
`C | F | C |` **`D 7`**`, G min 7 / F#7 | F …`with rule (Back Propagation of Seventh):
`C | F | C /` **`D 7`** `| G min 7 / F#7 | F …` with rule (Left Deletion):
`C | F | D 7 | G min 7 / F#7 | F …`

The same sequence of rules is applied on the `D 7` chord, with rules (Preparation by Minor Seventh), (Tritone Substitution), (Preparation by Seventh), (Back Propagation of Seventh) and (Left Deletion), to yield:

```
C | F / E 7| A min 7 / G# 7 | G min 7 / F#7 | F …
```

Finally the same rules are applied on the `E 7` chord to obtain the target chord sequence.

2.4.3 Example #2: *Turnarounds*

A *turnaround* is a typical small Jazz sequence of four chords, traditionally located at the end of a tune. Its function is to replace a first degree chord followed by its seventh (e.g. `C | G 7`). The simplest turnaround is probably the following:

```
(Turnaround #1)         C / A 7 | D min 7 / G 7
```

Many variations from this turnaround have been produced by Jazz composers. A nice example from Bill Evans is the following:

```
(Turnaround #2)         C / Eb 7 | Ab 7 / Db maj7
```

The first turnaround may be obtained simply by applying the following chord substitution rules, starting from `C | C| C` (3 measures):

`C |` **`C`** `| C`	with rule (Preparation with Seventh):
`C	` **`G 7`** `/ C	C`	with rule (Preparation with Minor):
`C	` **`D min 7`** `/ G7	C`	with rule (Preparation with Seventh):
`C	` **`A7`**`, D7 / G 7	C`	with rule (Back Propagation of Seventh):
`C / A 7	D min 7 / G 7	C`	QED.

The second one, however, is impossible to obtain from our chord substitution rules. The best approximation we can get is the following:

`C	C |` **`C`**	with rule (Preparation with Seventh):
`C	C |` **`G 7`** `/ C`	with rule (Back Propagation of Seventh):
`C |` **`G 7`**`| C`	with rule (Preparation with Seventh):
`C |` **`D 7`** `/ G7	C`	with rule (Preparation with Seventh):
`C |` **`A7`**`, D7 / G 7| C`	with rule (Back Propagation of Seventh):
`C /` **`A 7`** `| D 7 / G 7	C`	with rule (Tritone Substitution):
`C / D#7 |` **`D 7`** `/ G 7	C`	with rule (Tritone Substitution):
`C / D#7	G# 7 /` **`G 7`** `| C`	with rule (Tritone Substitution):
`C / D#7	G# 7 / C# 7| C`	which is equivalent to:
`C / Eb7	Ab 7 / Db 7| C`	

It is interesting to note that Bill Evan's turnaround cannot be reached in a proper way by applying the rules: one needs to polish by hand by replacing the last chord (`Db 7`) by a `Db maj 7` chord (we do not consider the problem of enharmonic spelling, i.e. the difference between `Eb` and `D#` here). The problem could be solved somehow by adding a rule such as `C 7` → `C maj7`, but doing so would create a lot of sequences which do not make sense in the context of Jazz. A more proper way to solve the problem is to introduce a "2 by 2" rule such as: `G 7 / C`

→ `Db maj7 / C` which does make sense musically (a kind of so-called *Neapolitan Sixth* rule in Classical music [3]).

With such a rule, the derivation would be quite different:

C | C | **C**	with rule (Preparation with Seventh):
C | C | **G7** / C	with rule (Back Propagation of Seventh):
C | **G7 | C**	with rule (Neapolitan Sixth):
C | **Db maj7** | C	with rule (Preparation with Seventh):
C | **Ab 7** / Db maj7 | C	with rule (Preparation with Seventh):
C | **Eb 7**; Ab 7 / Db maj7 | C	with rule (Back Propagation of Seventh):
C / Eb 7 | Ab 7 / Db maj7 | C	

which yields the right solution.

However, although such a rule makes sense, this shows the limit of a manually built rule set: how can one be sure that the set of rules is consistent, sound, or complete ? (see Section 3).

2.4.4 Example #3 : Exhibiting a Surprising Harmony

Now it is important to see that chord substitution rules, although they are all "licit" in themselves, can yield when combined together unexpected harmonies. Here is a simple example, starting from a simple `C 7` chord:

C 7	with rule (Preparation with minor):
G min 7 | C7	with rule (Tritone Substitution):
G min 7 | F#7	with rule (Preparation with minor):
G min 7 | C# min 7 / F#7	with rule (Preparation with seventh):
G min 7 | G#7, C# min 7 / F#7	with rule (Back Propagation of Seventh):
G min 7 / G#7 | C# min 7 / F#7	

What is surprising here is the appearance of a `C# min 7` chord in the context of `C 7`, which is, harmonically, quite out of the scope of the traditional harmonies supported by `C 7` (i.e. in Jazz, either `F`, `G` or `C`). However, the main claim of this paper is that the "surprise" is relative to the knowledge of the underlying chord substitution rules, and the ability to combined them in various ways.

To put it differently, an ear trained only by detecting patterns, i.e. recurring subsequences of data, would take much longer to accept this kind of sequence than an ear able to learn and use chord substitution rules. This clearly shows that learning Jazz harmonies involves more than learning simple patterns. The combinatorial aspect of Jazz harmony, formalized here as chord substitution rules, accounts for a large part in the perception of Jazz chord sequences.

2.5 The Harmonic Analysis problem

This description of the algebra of chord sequences raises a corresponding analysis problem: how to infer, from a given sequence and a rule set, a derivation tree that explains how the sequence may be derived from a basic, axiomatic sequence. This is the problem addressed in principle by Steedman, using a grammar-based approach but which was not solved

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[3] This rule was suggested by Marc Chemillier

operationally, since the corresponding exhibited grammar is context-dependent. We proposed in (Pachet, 1999) an approach to solve the problem, but argued that is not solvable in its full generality, even for standard sequences (for instance, we were not able to “prove” that the famous tune “Solar” by Miles Davis may be reduced to a basic Blues structure).

To summarize, we have identified two main ingredients for producing expectation and surprise in Jazz harmony: chord sequence *patterns*, and *chord substitution* rules. These ingredients are of course related: chord substitution rules produce corresponding chord patterns, and most of chord patterns may be inferred from chord substitution rules. However, they are not equivalent: chord patterns represent regularities in the data itself, predictable merely by their probability of occurrence, whereas rules represent normative musical knowledge, which - fortunately - is not applied systematically, but which nonetheless underlie most harmonic structures.

We will address these two problems separately using a statistical-based approach. For each of them we give some results of ongoing experiments. We will finally conclude on proposals to merge the two approaches towards a full model of gradual musical learning.

3. Extracting Patterns from Chord Sequences

(Dubnov et al, 1998) presented statistical analyses and re-generation methods based upon modern non-parametric techniques of string compression and comparison. These methods are capable of capturing long melodic structures, are easy to implement and have shown promising results for composition and style classification. We show in this Section how to apply these techniques for prediction of chord sequences, and use the model to model harmonic surprise.

3.1 Lempel-ziv applied to chord sequences

The Lempel-Ziv (LZ) data compression algorithm (Ziv & Lempel, 1978) uses an efficient one-pass pattern detection mechanism in order to build a dictionary of substrings. For the purpose of sequence generation, we can ignore the encoding part of the algorithm, and use only its pattern detection and representation scheme. In our experiments we used only chord sequences as input, ignoring the time dimension. The LZ parsing algorithm parses a sequence sequentially into distinct phrases, such that each phrase is the shortest string which is not a previously parsed phrase. From the Lempel-Ziv dictionary, we derive another representation, called LZ-tree. Each node in this tree represents a possible substring. The sons of the nodes represent the possible continuations of the substring. By construction, the number of sons is the probability of occurrence of the substring.

In order to use this scheme to make prediction and model surprise, we designed the following procedure: at each step, the sequence being built is compared to the tree. First the whole sequence is considered, and possible continuations are looked for. Then the process is iterated with the subsequence starting from the second element, and so forth until the last one. The result is a list of possible continuations sorted according to two criteria: 1) length of the subsequence and 2) weight of the continuations.

3.2 Expectation and surprise

Information theory yields a good definition of expectation. For sequences, the most expected item is obtained by taking the longest possible subsequence for which there is a possible continuation, and choosing the continuations with the highest probability. However, there is no such a simple canonical definition for surprise. There are several ways to define surprise in the context of our sequences: the simplest way is to define surprise as “the less expected item considering the shortest substring, i.e. the last element”. However, other definitions would be possible, such as: “the less probable item for the longest substring”, or any other choice within the list of possible continuation. In our experiments, we decided, by default, to choose the last item of our list, i.e. the less expected element considering the shortest substring. By definition, this element is not a possible continuation of any longer substring, so it yields a surprise which is unrelated to the past. We call the first element of this list called “`E`” (as most expected), and the last element “`S`” (as most surprising).

3.3 Learning chord changes instead of chords

Instead of learning chord sequences, we choose to learn sequences of chord changes. This “trick” allows to bypass the problem of transposition. Indeed, the two following sequences are equivalent, once transposed:

```
C | F maj7 | D min 7 / G 7
E | A maj7 | F# min 7 / B 7
```

However, it is difficult to normalize chord sequences since this would require the knowledge of the tonality of a chord sequence. As explained in Section 2.5, extracting the tonality requires an harmonic analysis, which is a very difficult problem. Moreover, Jazz chord sequences contain a lot of modulations (changes in tonality) so this transposition would solve the problem only locally for small segments of a sequence.

To solve this problem, we instead propose to learn sequences of chord *transition*. A chord transition, in our context, is a couple of chords transposed in `C`. It can be represented as a couple of chords whose first chord is in `C` (`Chord1 : Chord 2`). For instance, the chord transition sequence corresponding to:

```
E | A maj7 | F# min 7 | B 7
```

is the following:

```
(C : F maj7) | (C maj7 : A min 7) | (C min 7 : F 7)
```

The Lempel-Ziv tree represents therefore the possible continuations of a given chord transition, or chord transition subsequence.

3.4 Implementation and Validation

In this section we show that this learning mechanism produces correct notions of “surprise” and expectation are achieved rather quickly by learning chord transitions. The first learning corpus is the set of 4 Charlie Parker chord sequences of Figure 2 (*BluesForAlice, Marmaduke, NowsTheTime, Ornithology*).

3.4.1 The LZ tree of chord transitions

The corresponding LZ tree built from the corpus is the following. Note that the first chord of all chord transitions is C. Indentation reflects the hierarchical structure.

```
C :C
---C :C  min
------C  min:F  7
C :C  min
---C  min:F  7
```

```
C  min:F  7
---C  7:F  7
---C  7:F  min
------C  min:F  7
---------C  7:F  7
------C  min:C  min
---C  7:F
------C :D  min
---------C  min:F  7
------C :E  min
------C :C
---------C :A  7
---C  7:G  min
---C  7:F#  min 7
C  7:F
---C :C
------C :E  min
---C :A  7
---C :D  min
---C :E  min
C  7:F#  halfDim7
---C  halfDim7:F  7
C  halfDim7:F  7
---C  7:F  min
C  7:F  min
---C  min:F  7
------C  7:F  7
C  min:G  aug9
C  aug9:A  min
C  7:F  7
---C  7:D  min
C  min:C  min
---C  min:C  min
------C  min:F  7
------C  min:C  min
C :G min
C  7:C  7
---C  7:C  7
---C  7:F  7
---C  7:G  7
C  7:C  min
---C  min:F  7
C  7:A  7
C  7:G  7
C :B  halfDim7
C  7:F#  min
C  min:Cb  min
C  min 7:F  7
```

Figure 3. The LZ tree of chord transitions.

3.4.2 Surprise and expectation

To illustrate how this model can produce expectations and surprise, we will consider the starting sequence: `C min | F 7.` Here is the list of sorted possible continuations given by the sorted LZ-tree for this sequence:

```
 -- past size = 1
C  7:F  min
C  7:F  7
C  7:F
C  7:F#  min 7
C  7:G  min
 -- past size = 0
C  7:C  7
C  7:C  min
C  7:G  7
C  7:F#  halfDim7
C  7:F#  min
C  7:D  min
C  7:A  7
```

Figure 4. Possible continuations after a given subsequence, as given by the learning system. The most expected is `C 7: F min` - which yields `Bb min` in our context - the most surprising is `C 7:A 7`; which yields `D 7` here.

By construction, the possible continuations of size I are not repeated for size `j<i`.

3.4.3 Examples of generated sequences

We will now produce sequences according to two different schemes of surprise and expectations from the starting sequence and the learned LZ-tree. The first one is a series of "most expected" chords only (represented as the sequence "`e e e e e e e`"). The second one is a series of "most expected" with two surprises inserted ("`e s e e s e e`"). Note that the first two chord are represented as only one "e" since the systems knows only about chord transitions.

Example #1, only most expected chords:

```
C min 7 | F 7 | Bb min | Eb 7 | Ab 7 | Bb min | Eb 7 | Ab min
          e     e        e      e      e        e      e
```

Example #2, introducing surprise:

```
C min | F 7 | D 7 | G min | C 7 | A 7 | D min | G 7
        e     s     e       e     s     e       e
```

The corresponding harmonic progressions effects are indeed quite satisfactory, musically speaking: the system has "learned" about *two-five-one* transitions, and is able to complete sequences by resolving seventh chords. The surprise (transition from `F 7` to `D 7`) is of course not very surprising for a trained ear but quite novel considering this stage of learning.

After learning the whole corpus of chord sequences, the results are the following:

Example #1:

```
C min | F 7 | Bb | C min | F 7 | F min | Bb 7 | Eb

        e     e    e       e     e       e      e
```

This result shows that the system is able to distinguish between two occurrences of the same chord depending on the past (`A min` at the beginning, and at 4th position).

Introducing some surprise in the sequence yields:

Example #2:

```
C min | F 7 | F# min 7 | B 7 | E 7 | F min 7 | Bb 7 | Eb
        e     s          e     e     s         s      e
```

The "surprise" learned by the system consists in a rather untypical harmonic progression from a seventh chord to its sharpened minor seventh (`C 7 : C# min 7`), which could be considered as "a pleasant surprise" by trained Jazz ears.

These experiments show that the system is able to quickly learn Jazz chord patterns, and create expectations in accordance with the theory, based on these patterns.

3.4.4 Limitations of the surprise ability

However, although the system learns patterns of chord changes, it still has a limited capacity to be surprised: any unexpected chord is surprising, since the system has no knowledge on the underlying combinatorial algebra of chords.

For instance, suppose that the system has learned about resolution of seventh patterns, and the resolution of Tritone chords. In this case, consider the following sequences:

`C | C7 | F` considered not surprising (resolution of seventh pattern already known)

`C | F#7 | F` considered not surprising (resolution of tritone pattern already known)

`C | C# - 7 / F#7 | F` considered as very surprising, since, in this case, the transition of `C` to `C# min 7` has not yet been heard, or very rarely.

However, an agent who would "know" about the (Preparation by Minor Seventh) rule would somehow be able to understand the last sequence and fall back on its "pattern base" by the following reasoning:

`C | C# - 7 / F#7 | F` is equivalent to:

`C | F#7 | F` by application of the (Preparation by Minor Seventh) rule which is known.

This example shows the limitation of the purely pattern-based approach for modeling surprise. In the next section we propose a mechanism to learn chord substitution rules automatically from the analysis of a corpus.

4. Extracting Substitution Rules from Chord Sequences

In this section we will examine how to induce from the corpus chord substitution rules as described in Section 2.4.

4.1 The Rule inference Model

In Section 2, we emphasized the fact that many chords in a Jazz sequence are obtained by applying rewriting rules to other chords or chord sequences. A characteristic of these rules is their "local" aspect: rule applications affect only a chord or group of chords and their immediate neighbors. As a consequence, we can assume that a rule rewriting will somehow preserve neighbors.

Thus we can define a (log)likelihood for a rewrite rule `R = (A→B)` to happen as `<log(P(R|X))>`, where <> signifies averaging with respect to `X` and `P(R|X)` is the a posteriori probability of `R` given `X`.

From definition of KL distance (cross entropy) it follows that:

`<log(P(R|X))> = <log (P(X|R)/P(X))> + log(P(R)) = D(X|R),P(X)) + log(P(R))`

Ignoring the prior `P(R)`, we say that the likelihood of occurrence of the rule `R` is equivalent to the similarity in distribution of the data `X` before and after the application of the rule `R`. Now we make another simplification and approximate `X` by the pattern `LAR` for given chord before application of the rule and the corresponding pattern `LBR` after application of the rule.

Thus the likelihood for rule `R` can be represented as the decision `D(P(LAR), P(LBR)) > Threshold`. Calculating this `D(P(LAR), P(LBR))` is easy. For a given chord `A` we construct the table `Pa(LR) = P(LR|A)` of probabilities to see the neighbors `L` and `R` around `A`: `P(LAR) =`

Pa(LR) * P(A). P_a(LR) is constructed as a matrix with the entries L, R. At each entry we write down the number of occurrences of the pattern LAB in the corpus. For example, consider the following sequence:

```
C | A | F | G | C | C | F | G | C | C | F | A | G | C | A | F
```

We construct a table with the following entries:

For chord A we have $P(A) = 3/16$ (length of the sequence) and P_a is:

```
C, F -> 2/3
F, G -> 1/3
```

and for chord C, P(C) = 6/16 and P_c is:

```
G, C -> 2/5
C, F -> 2/5
G, A -> 1/5
```

Now for every candidate rule A→ B we calculate D(P_a*P(A), P_b*P(B)), by summing over all entries L, R the distortion contribution of LR:

$$D(P_a * P(A), P_b * P(B)) = \sum_{LR} P_a(LR) * P(A) * \log\left(\frac{P_a(LR) * P(A)}{P_b(LR) * P(B)}\right)$$

Note that this is an unsymmetrical quality and thus contains the direction of A → B. In probabilistic terms, this means that given the distribution of LAR, the probability to see the sequence LBR is proportional to $e^{(-ND)}$ where N is the number of samples (length of the sequence).

Finally, we extend this formula to handle transpositions: each transition matrix P_a(LR) records the transposed occurrence of the context (in C). This requires a small trick in calculating the sum so that transpositions are done in reverse when looking up a context LR from A to B.

4.2 Practical experiments

The experiments reported here consisted in extracting four kinds of chord substitution rules, according to the formula above, corresponding to 1-1, 1-2, and 2-2 rules. To avoid problems with infinite quantities when an item B does not appear in a context (LR) of A, we arbitrarily assign a low probability to unseen events (10^{-4}).

The procedure consists then in taking each possible item (either chord or sub sequence), and computing all possible candidates substitutions, and sort them according to the distortion measure. Only the two best items are given.

4.3 Evaluation

This section gives the result of our rule extraction procedure applied on a corpus of 76 Jazz chord sequences (52 tunes by Charlie Parker, plus 24 standard tunes taken out of the Real Book including the ones in Figure 2). We give for each rule a few comments on its relevance.

4.3.1 Chord substitutions 1 to 1

62 rules involving 13 chord types were induced. We indicate here for each chord type only the three best rules (or less if there were only less rules found).

C halfDim7 --> F 7
C halfDim7 --> C min

C halfDim7 --> C min 7

C aug5 7 --> F 7
C aug5 7 --> G min

C aug5 7 --> C 7

C min 9 --> C 9
C min 9 -->C 7
C min 9 -->C min 7

C --> C min
C --> G 7
C --> C 7

C min 7 --> F# 7
C min 7 --> C 7
C min 7 --> C min

C maj7 --> C 7

C maj7 --> C min 7
C maj7 --> C

C dim9 --> D#
C dim9 --> F min
C dim9 --> C 7

C dim7 --> B 7
C dim7 --> A# dim7
C dim7 --> F# 7

C aug9 --> Bb 7
C aug9 --> F min 7
C aug9 --> C 7

C min --> A#
C min --> F 7
C min --> C min 7

C 9 --> Bb 7
C 9 --> F min
C 9 --> C 7

C 7 --> G 7
C 7 --> F 7
C 7 --> G min

C 13 --> G min 7
C 13 --> C 7

All these rules are "correct", i.e. make sense musically and can be explained in terms of the basic rules described in Section 2.4. Some of these rules are either enrichments or simplifications of chord structures (e.g. `C aug5 7 → C 7`). Other rules contain a "Tritone Substitution" flavor (e.g. `C min 7 → F# 7`), sometimes combined with another simplification rule (`C aug9 → F# min 7`). Some rules contain an incomplete preparation by minor (e.g. `C 7 → G min`). Going up and considering rules with a higher distortion rate, one finds rules such as: (`C 7 → G# dim7`) which do not make much sense. These rules are probably due to the size of the training set, and should disappear on full sets of Jazz corpuses. However, limiting the output to only the best rules yields an almost "perfect" result.

4.3.2 Chord substitutions 1 to 2

These rules are of the form: one chord → two chords. 847 rules were found. We list here only the three best rules for the most common chord types.

C --> [G 7; C]
C --> [C ; C]
C --> [D min; G 7]

C min 7 --> [C 7; C min 7]
C min 7 --> [F ; C min]
C min 7 --> [G min; C 7]

C dim7 --> [G# 7; C# min]
C dim7 --> [E 7; E 7]
C dim7 --> [B min; E 7]

C aug9 --> [E min; A 7]
C aug9 --> [C# ; C#]
C aug9 --> [D# min; G# 7]

C min --> [A# ; C min]
C min --> [C 7; C min]
C min --> [G min; C 7]

C 9 --> [A# 7; D# 7]
C 9 --> [F min; F min]
C 9 --> [A# 7; D#]

C 7 --> [C 7; F]
C 7 --> [G min; C 7]
C 7 --> [C 7; F 7]

C halfDim7 --> [F# 7; G#]
C halfDim7 --> [C# ; C min]
C halfDim7 --> [C# ; F 7]

We can notice that the rules induced contain the most common chord substitution described in Section 2.4. For instance, the (Preparation by Seventh) rule, instantiated for major chords (`C → G7 / C`), the (Preparation by Minor) (C 7 → G min 7 / C 7), the (Repetition) rule (C →C / C), the (Transition to Fourth) rule (`C7 → C7 / F`). Other rules are induced which are not listed explicitly in our rule set, but which are either combinations of rules (e.g. `C → D min 7 / G 7` can be seen as a combination of `C → G7 / C` and `G 7 → D min 7 / G 7` yielding `C → D min 7, G 7 / C 7`, and then truncated). Finally new rules are also found such as: `C 9 → A# 7 / D#`, which can be explained musically (the `aug9` of `C` is indeed `D#`, which is then "prepared" by its seventh `A# 7`).

4.3.3 Chord substitutions 2-to-2

These rules are of the form: `X Y → Z T`. 786 rules involving 198 different subsequences types (i.e. couples of two consecutive chords) were found. For reasons of space, we indicate here only the most interesting rules. We classify the rules in six groups: rules dealing with equivalence of a single chord in a given context (diatonic or non diatonic), rules with a tritone substitution, rules dealing with preparations (minor or seventh), rules dealing with various kinds of two-five structures, and interesting rules (i.e. rules with non trivial musical meaning).

Equivalences

Diatonic equivalences
```
[C  7; A  7] → [E  min; A  7]
[C  7; G# ] → [C  7; C  min]
[C  7; F# ] → [E  maj7; A#  min 7]
[C 7; B min 7] → [G maj7; B min 7]
[C 7; A  min 7] → [C  7; F  maj7]
```

Non diatonic equivalences
```
[C 7; E min] → [A# maj7; E  min 7]
[C maj7; A min 7] → [D# maj7; A min 7]
[C maj7; F# min 7] → [C# min 7; F# 7]
[C7; B halfDim7] → [G min 7; A# min 7]
[C  maj7; C maj7] → [C ; D# dim7]
[C aug9; F min 7] → [A#7; F min 7]
[C  7; G#  min] → [E ; G# min]
[C ; D# min] → [A maj7; D# min 7]
[C  7; F ] → [C  7; F 7]
```

Tritone flavor
```
[C ; G  min] → [F  min; A#  7]
[C ; G  min 7] → [F  min; A#  7]
[C maj7; F# min9] → [C# min7; F# 7]
[C 7; C#  min7] → [F# ; C# min]
[C min7; B min7] → [F# aug9; B min 7]
[C  min; B ] → [C  min; F  7]
[C  7; F  min] → [F#  min; F  min]
[C ; B ] → [E  7; F ]
```

Preparations
```
[C ; F  7] → [C ; C  min]
[C ; E  7] → [C ; B  min]
[C ; A  min 7] → [E  7; A  min]
[C ; D  7] → [A  7; D  7]
[C ; C  7] → [G  min 7; C  7]
[C 7; B ] → [F#  min; B  7]
[C 7; B min] → [F# 7; B  min 7]
[C min; G min] → [F# dim7; G  min]
[C maj7; B min 7] → [F#7; B min 7]
[C ; G#  min] → [D#  min; G#  7]
```

Two fives
```
[C ; C ] → [D min 7; G  7]
[C min; C min] → [D  min; G 7]
[C min7; D# min 7] → [F 7; E halfDim7]
[C min 7; A# min 7] → [F 7; A# ]
[C min 7; C min 7] → [F7; C min 7]
```

Simplification
```
[C min; F dim9] → [C ; F  7]
[C maj7; E min 7] → [C maj7; A 7]
[C  maj7; C  min 7] → [C ; C ]
[C  13; C  13] → [C  7; C  7]
```

Interesting rules
```
[C ; C# ] → [C# ; C ]
[C  min; G  7] → [G  min; C  7]
[C  7; B  7] → [D ; F  dim7]
[C  9; F  min] → [A#  7; D# ]
[C  min; F  7] → [A# ; A# ]
[C ; B  7] → [E  7; B  7]
[C ; D#  dim7] → [C  maj7; A  7]
[C min 7; C# min 7] → [D# 7; G# ]
[C ; F# 7] → [A min 7; D  7]
```

Here again, the output shows that most of the rules described in 2.4 are "captured", though in a form which may differ from the canonical versions, but which is extracted automatically. An important point is that all the rules make sense: the approach extracts significant information.

4.3.4 Chord substitutions 2 to 1

2-1 rules (of the form: two chords → one chord) were also computed but are not reported here because they are less “natural” in harmony (we found 1518 of these rules).

5. Conclusion

The studies described in this paper address the modeling of harmonic surprise from a computational viewpoint. First we showed that it is possible to extract pattern harmonic information on chord sequences automatically in an unsupervised manner, using simple data compression techniques. Experiments on Jazz harmony show that the extracted patterns provide a satisfying notion of expectation and surprise, which capture important regularities of Jazz harmony. Second, we have argued that the underlying algebra of Jazz harmony, represented as chord substitution rules, plays an important role in the perception of chord sequences, in that they allow to understand more sequences than what is allowed by the mere analysis of recurring patterns. These rules are difficult to learn in theory because they involve studying all possible contexts of subsequences, but we propose an approximation to contexts limited to only one neighboring chord. We show that we are able to induce a number of rules from the gradual analysis of the corpus, and that the rules induced do correspond, in general, to the usual chord substitution rules of music theory textbooks, plus many others.

These two experiments form the basic blocks for a complete model of gradual musical learning yet to be designed. Such a model could explain not only how we gradually learn new sequences, but how we gradually learn how to learn new sequences. The underlying motivation of this work is that surprise may be related to calculus, or, in our algebraic context of jazz harmony, to proof: in this respect, a chord sequence would be surprising to the extent that it is “provable” by the hearer, more than to the extent that it has already been heard before. The two ingredients (corpus of already heard patterns, and set of rules) are necessary, and this paper shows that they can be modeled successfully independently.

Ongoing work focuses on building a computational model of musical memory that accounts for this double facility to identity recurring patterns and induce substitution rules. In particular, one important effect of learning rules is that this allows to reduce the memory needed for recording patterns (in our case, the LZ-tree). Such a model would further allow to study the effect of size limitation constraints on the memory: in this view, to learn more, the system would have to induce rules - produce abstractions - in order to make room for new patterns and rules to be learned. To come back to the introductory quotation of Ernst Gombrich, we believe that a finer model of harmonic expectation should bring a better understanding of musical surprise, hence of the understanding of aesthetic pleasure.

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